ablator surface. The highly reflective surface would receive the radiant heating peak of the superorbital re-entry and give way to another high-emissivity layer as q_{Ri}/E_A decreased through one.

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Real-Gas Hypersonic Blunt-Body Flows

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Introduction

SEVERAL accurate methods have been developed for analyzing inviscid hypersonic flows around blunt bodies (see, e.g., Refs. 1–3). In most of these methods, however, the gas is assumed to be perfect, i.e., one that obeys the equation of state $p = \rho RT$, where p is the pressure, ρ the density, R the gas constant, and T the temperature, and has a constant ratio of specific heats. The use of such a state equation is well justified in that accurate results are obtained at flight Mach numbers for which molecular vibration, dissociation, and ionization in the flow between the detached bow shock wave and the body is nonexistent or negligibly small. However, as the flight Mach number and hence temperature behind the shock increases, the vibrational energy modes become excited and the gas begins to dissociate and ionize. Upon excitation of the vibrational energy modes, the specific-heat ratio no longer remains constant, and, with the onset of dissociation, the relation $p = \rho RT$ becomes inaccurate. Thus, although perfect-gas results for flight Mach numbers above those at which real-gas effects become significant still yield important qualitative features of the flow field, quantitative accuracy of thermodynamic and physical variables decreases.

In order to increase the accuracy of existing inviscid equilibrium perfect-gas blunt-body solutions at flight Mach numbers above those at which real-gas effects become important, several investigators have modified perfect-gas solutions to handle real-gas effects. The Research and Advanced Systems Branch of the Aero and Propulsion Sciences Group at Norair Division of the Northrop Corporation has developed an equilibrium real-gas solution[4] in which the perfect-gas equation of state is replaced by Hansen's[5] closed-form expressions for the thermodynamic properties of equilibrium air. Van Dyke's[6] method of numerical integration of partial differential equations is used by the Northrop group. Lomax and Inouye[7] of NASA-Ames have developed a solution combining the equilibrium-air data of Hilsenrath and Beckett[8] with the Van Dyke Solution.†

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† This work of Lomax and Inouye is unpublished to date. Some of the results were made available to the author and are used in this note for comparison purposes.

For real-gas solutions such as developed at Northrop and Ames, a substantial amount of labor is required to incorporate the real-gas equations of state into the basic solution. Hansen's expressions for the thermodynamic properties are lengthy, and the NASA solution requires computing machine storage of the Hilsenrath and Beckett tables, along with table look-up and interpolation routines. Hence, it would be advantageous if an approximate equilibrium real-gas thermodynamic description could be developed that, when used to replace perfect-gas thermodynamics in any accurate blunt-body solution, would yield accuracy to within a few percent of the more exact solutions at only a fraction of the labor.

By assuming the equilibrium real-gas flow between the shock and a blunt-nosed body to be one of constant specific-heat ratio different from the undissociated freestream value, an approximate equilibrium real-gas thermodynamic description is obtained. The author's blunt-body method[9] is modified to accommodate the approximate thermodynamic description of the actual equilibrium real-gas flow. Results are compared with those obtained at Northrop and Ames, and also with perfect-gas solutions.

Approximate Equilibrium Real-Gas Thermodynamics

As previously mentioned, the equilibrium real-gas flow between the shock wave and body is assumed to be one of constant specific-heat ratio. In addition, this gas is assumed to obey the equation of state

$$h = [\gamma/(\gamma - 1)](p/\rho) + \tilde{A} \tag{1}$$

where h is the enthalpy, p the pressure, $\tilde{A}$ a constant, and γ the adiabatic index defined by

$$\gamma = (\partial \ln p/\partial \ln \rho)_s = a^2\rho/p \tag{2}$$

where s is the entropy and a the speed of sound. The assumption of Eq. (1) for the equation of state in the shock layer can be given physical justification by noting that it is tantamount to assuming that each species in the shock layer behaves as a perfect gas having the same ratio of specific heats, γ. The constant $\tilde{A}$ is then identifiable with the dissociation-energy contribution to the enthalpy.

By use of the Second Law of Thermodynamics it can be proven that, for a gas having the state equation, Eq. (1), the entropy s is a function of p/ρ^γ only. Thus, since entropy is conserved along streamlines in equilibrium flow, introduction of a stream function into the governing equations of fluid mechanics yields

$$p/\rho^\gamma = g(\psi) \tag{3}$$

where ψ is the stream function and the functional form of g is determined from the boundary conditions at the shock wave. Then, modification of an accurate blunt-body solution such as that of Van Dyke[1] or Swigart[9] reduces to modification of the shock-wave boundary conditions and the function $g(\psi)$. For the real-gas description under consideration, the density ratio across the shock wave becomes

$$\frac{\rho_s}{\rho_\infty} = \left\{\frac{\gamma}{\gamma+1}\left[1 + \frac{1 - B\xi^2}{\gamma_\infty M^2(1 + C\xi^2)}\right] - \frac{\gamma}{\gamma_\infty(\gamma+1)M^2} \times \left[\left(\frac{1 + C\xi^2}{1 - B\xi^2}\right)^2 + \left(\frac{\gamma_\infty}{\gamma}\right)^2 M^4 \left(\frac{2\gamma^2}{\gamma_\infty M^2} - 2(\gamma^2 - 1) \times \left\{\frac{1}{(\gamma_\infty - 1)M^2} - A\right\}\right)\left(\frac{1 + C\xi^2}{1 - B\xi^2}\right) + \left(\frac{\gamma_\infty}{\gamma}\right)^2 M^4\right]^{1/2}\right\}^{-1} \tag{4}$$

where B is a parameter characterizing the shock-wave shape,[6,9] $C = 1 - B$, ξ is distance along the shock, M is the freestream Mach number, $A = \tilde{A}/q_\infty{}^2$, where q_∞ is the freestream speed, and γ_∞ is the specific-heat ratio in the freestream. The derivative of the stream function normal to the shock immediately behind the shock is given by

$$\left.\frac{\partial \psi}{\partial \eta}\right|_s = \frac{\rho_s}{\rho_\infty}\xi^2 \tag{5}$$

and the function of $g(\psi)$ is determined to be

$$g(\psi) = \frac{\dfrac{1}{\gamma_\infty M^2} + \dfrac{1 - 2B\psi}{1 + 2C\psi}\left(1 - \dfrac{1}{D}\right)}{D^\gamma} \tag{6}$$

where

$$D = \left\{\frac{\gamma}{\gamma + 1}\left[1 + \frac{1 - 2B\psi}{\gamma_\infty M^2(1 + 2C\psi)}\right] - \frac{\gamma}{\gamma_\infty(\gamma + 1)M^2} \times \left[\left(\frac{1 + 2C\psi}{1 - 2B\psi}\right)^2 + \left(\frac{\gamma_\infty}{\gamma}\right)^2 M^4 \times \left(\frac{2\gamma^2}{\gamma_\infty M^2} - 2(\gamma^2 - 1)\left\{\frac{1}{(\gamma_\infty - 1)M^2} - A\right\}\right) \times \left(\frac{1 + 2C\psi}{1 - 2B\psi}\right) + \left(\frac{\gamma_\infty}{\gamma}\right)^2 M^4\right]^{1/2}\right\}^{-1} \tag{7}$$

For details of the derivations of these equations, and the modification of the perfect-gas blunt-body solution, see Swigart.[10]

Determination of γ

Since the gas between the shock and body is assumed to have a constant ratio of specific heats γ, and the governing differential equations and boundary conditions at the shock wave depend parametrically on γ, a value for this quantity must be determined before the flow field can be obtained.

Use of the conservation equations in integral form, along with state data in tabular or graphical form enables a precise determination of γ at any point immediately behind the shock and at the stagnation and sonic points on the body. Use of the approximate state equation, Eq. (1), then enables corresponding values of the constant A to be determined at these points. For the cases investigated, the assumed constant values of γ and A in the shock layer were taken to be the average of those four values determined by using the conservation equations in integral form, a Mollier diagram for equilibrium air,[10] and Eq. (1) immediately behind the shock along the stagnation streamline, at the stagnation point on the body, and at the sonic points immediately behind the shock and on the body.

Results and Discussions

In order to compare the results of the present method with those of the more exact analyses[4,7] previously discussed, solutions were obtained for flow past spheres at altitudes of 100,000, 200,000, and 300,000 ft at freestream Mach numbers M of 12 and 25. Results for the Mach 25, 200,000 ft case are compared with those of Northrop/Norair and NASA-Ames in Figs. 1 and 2. Figure 1 compares results of the present method with those of Norair for body shapes, sonic lines, and the shock wave supported by a sphere in each case. Figure 2 compares the same quantities with the NASA-Ames results. Note that the shock wave supported by a sphere as determined by the present method begins to deviate from those of the more exact methods at a point beyond the sonic point on the shock wave, but still in the region where the right-running characteristics emanating from the shock wave intersect the sonic line and hence affect the subsonic region. Sonic line agreement is good, and the stand-off distance is slightly larger (3.5%) than that of the Norair solution, but agrees very well with the NASA-Ames result. Similar results are obtained for the other cases.[11]

In order to assess the relative effects of having a freestream ratio of specific heats different from the constant value selected in the shock layer, and of having a value of the constant, A, other than zero, the solution of the present method at Mach 25, 200,000 ft altitude is compared in Fig. 3 with two perfect-gas solutions and with a solution for which the specific-heat ratios in the shock layer and freestream are different, but A is zero. These comparisons yield some interesting results. Note that the solutions coincide to the scale of the plot for a perfect gas having a ratio of specific heats constant at the value in the shock layer (and, of course,

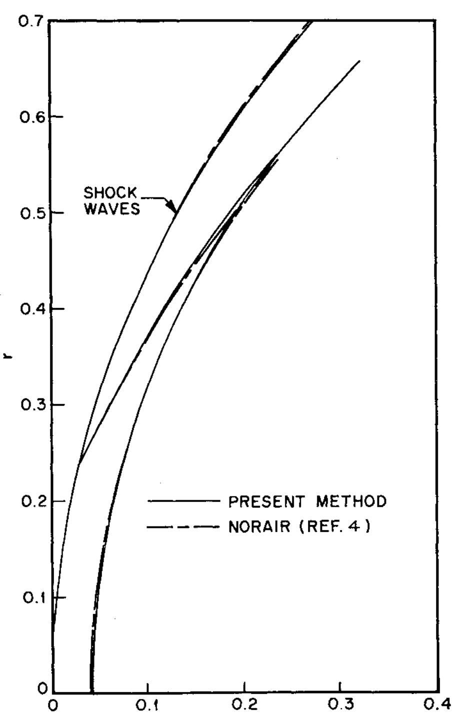

Fig. 1 Comparison of body shapes and sonic lines; M = 25, z = 200,000 ft.

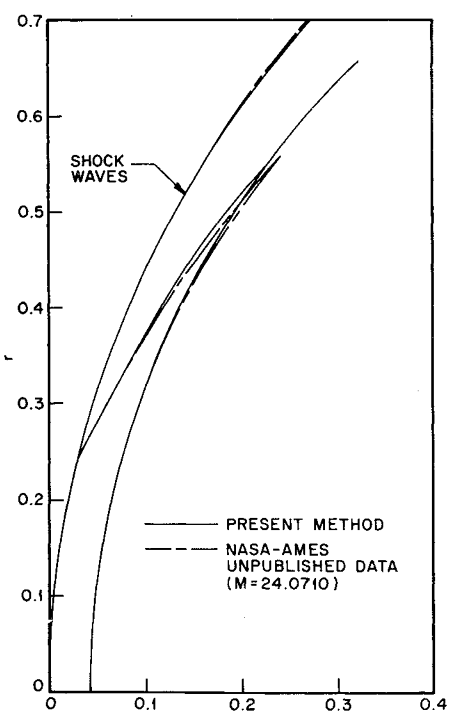

Fig. 2 Comparison of body shapes and sonic lines; M = 25, z = 200,000 ft.

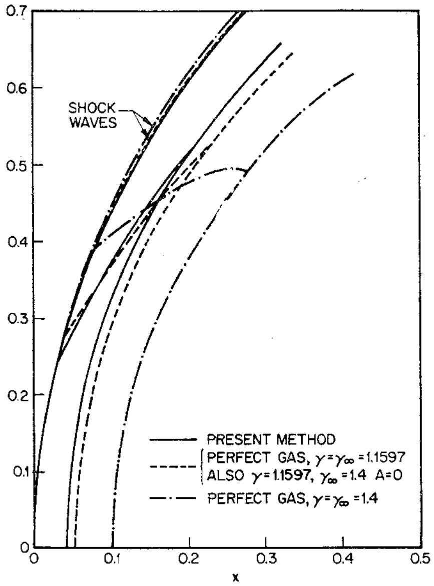

Fig. 3 Comparison of body shapes and sonic lines; $M = 25$, $z = 200{,}000$ ft.

$A = 0$), and for a specific-heat ratio in the shock layer different from the freestream value, again with $A = 0$. Further note the large difference between these coincident solutions and the solution for a perfect gas having a ratio of specific heats equal to the freestream value, and the difference between the coincident solutions and that of the present method. From these comparisons one may conclude that the main parameters affecting the solution are the ratio of specific heats in the shock layer and the constant A, whereas the value of the freestream specific-heat ratio has relatively little affect. Considering now the perfect-gas solution having $\gamma = 1.1597$ and the solution by the present method, one notes that, even though the shock waves supported by a sphere in both cases are coincident, the body and sonic-line positions are significantly different. Hence, large errors can be incurred in approximating a real-gas flow by a perfect gas having a constant ratio of specific heats corresponding to an average value of the actual flow in the shock layer. Even larger error is incurred by assuming the specific-heat ratio to be constant at the freestream value. However, the error is seen to be small in approximating a real-gas flow by one having a constant ratio of specific heats different from that of freestream, along with an appropriate value of the constant A as determined by Eq. (1) and the relation $A = \tilde{A}/q_\infty{}^2$.

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Heat Transfer to a Sphere for Free Molecule Flow of a Nonuniform Gas

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Theoretical relations are obtained for the heat transfer to a sphere for the free molecule flow of a nonuniform gas. The sphere diameter is assumed to be small compared to the molecular mean free path, and the gas flow is assumed to have arbitrary viscous stress and heat flux terms present, such as the case encountered in a boundary layer. The effect of these nonuniformities on the heat transfer to the sphere and its equilibrium temperature are determined theoretically.

Introduction

THE purpose of this analysis is to obtain theoretical relations for the heat transfer to spheres in a free molecule flow of a nonuniform gas. A nonuniform gas is defined as one whose distribution function deviates from the Maxwellian equilibrium distribution because of the presence of viscous stresses and heat flux terms. This problem arose in connection with free molecule spherical probes used for surveying boundary layers in supersonic low-density nozzles[2] as a direct extension to similar University of California free molecule cylindrical probes.[1] The method of approach closely parallels the analysis used by Bell and Schaaf[1] in the case of cylinders.

Procedure

The energy balance for a differential area dA in the absence of radiation is written as

$$dQ = dE_i - dE_r \tag{1}$$

where dE_i is the incident energy flux, dE_r the re-emitted energy flux, and dQ the net heat loss per unit time from the surface element. The thermal accommodation coefficient is defined as

$$\alpha = (dE_i - dE_r)/(dE_i - dE_w) \tag{2}$$

where dE_w is the energy flux that would be re-emitted from the surface if all molecules were re-emitted with a Maxwellian distribution corresponding to the surface temperature T_w. Introducing this definition in Eq. (1), one gets

$$(1/\alpha)dQ = dE_i - dE_w \tag{3}$$

The incident energy flux per unit area is

$$\frac{dE_i}{dA} = \frac{m}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} (u'^2 + v'^2 + w'^2)u'f du'dv'dw' \tag{4}$$

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